

## Note

These slides were used for a *hybrid* presentation. I performed live annotations on my tablet while presenting, so these slides are **not intended to be read on their own**.

A full text version of this talk will soon be available at:

[leon-schropp.pages.dev](https://leon-schropp.pages.dev)

You can also find an Abstract on the next slide!

Efficiently implementing universal logical gates remains a primary bottleneck for fault-tolerant quantum computation. Codes with native magic states offer a promising pathway to overcome this challenge. Guanyu Zhu (IBM) developed a geometric method for constructing such qLDPC codes: he maps general LDPC codes to manifolds, then retracts them back to low-overhead codes with transversal (non-)Clifford gates suited for near-term hardware. I'll briefly introduce this method before covering two results it has enabled.

- ➊ It yields qLDPC codes with transversal non-Clifford gates that break the long-standing Bravyi–König distance barrier: constant-rate codes reach distance  $\Omega(\sqrt{N})$ , and pushing further gives distance  $\Omega(N^{2/3})$  at the cost of a shrinking rate.
- ➋ Extending the method to non-Abelian codes yields addressable CZ magic states on ordinary constant-rate 2D product codes, via a gauging measurement protocol that sidesteps the usual need for 3D constructions.

Both approaches generally produce qLDPC codes with *magic state fountains*, enabling many disjoint, high-fidelity magic states to be prepared in parallel with low routing overhead and no distillation.

# Native Magic States in qLDPC codes.

From codes to manifolds and back

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G.Zhu: A topological theory for qLDPC. (arXiv:2501.19375)

G.Zhu: Transversal non-Clifford gates on qLDPC codes.  
(arXiv:2507.15056)

G.Zhu: Non-Abelian qLDPC (arXiv:2601.06736 )

## ✂ Magic State distillation (MSD)

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**Bravyi-König:**  $d \in O(n^{1/3})$



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## KYP-Unfolding

Let  $\mathcal{L}^D$  be a  $D$ -dimensional,  $(D + 1)$ -valent lattice with  $(D + 1)$  colors. Then,

- 1  $UCC(\mathcal{L}^D)U^\dagger \cong TC(\mathcal{L}^D)^{\otimes D}.$
- 2  $\overline{C^{D-1}Z_{\text{total}}}$  on  $TC(\mathcal{L}^D)^{\otimes D}$  corresponds to transversal gate in  $CC(\mathcal{L}^D).$

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## Pezzana Crystallization Theorem

Every  $D$ -dimensional PL-manifold admits a triangulation, which is a  $(D + 1)$ -valent lattice with  $(D + 1)$  colors.

Recall: Lie algebra  $\xrightarrow{\text{exp}}$  Lie Group

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$$Z = \exp(i\pi z) = (-1)^z \\ \text{for } z := \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

$$X = \exp(i\pi x) = (-1)^x \\ \text{for } x := \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{pmatrix}.$$

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## Gauge-Fields

Let  $\mathcal{C}_\bullet \in \text{Ch}(\mathbb{Z}/2)$  be a CSS code.

$$a: \mathcal{C}_1 \rightarrow \mathfrak{u}(2^n)$$

$$a(e_j) := z_j := l_2 \otimes \cdots \otimes l_2 \otimes z \otimes l_2 \otimes \cdots \otimes l_2.$$

$$b: \mathcal{C}_1 \rightarrow \mathfrak{u}(2^n),$$

$$b(e_j) := x_j = l_2 \otimes \cdots \otimes l_2 \otimes x \otimes l_2 \otimes \cdots \otimes l_2$$

$$\Rightarrow a, b \in C^1(\mathcal{C}_\bullet, \mathfrak{u}(2^n)).$$

Let  $\gamma = (\gamma^1, \dots, \gamma^n) \in \mathcal{C}_1 = (\mathbb{Z}/2)^n$ .

$\mathcal{C}_\bullet \in \text{Ch}(\mathbb{Z}/2)$  CSS  
code.

**Paulis:**

$$Z = (-1)^z,$$

$$X = (-1)^x$$

**Gauge-Fields:**

$$a(e_j) = z_j, b(e_j) = x_j$$

Let  $\gamma_1, \gamma_2 \in \mathcal{C}_1$ .  
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Let  $\mathcal{L}$  be a simplicial complex.

## Cup-Product

For  $\alpha^p \in C^p(\mathcal{L})$ ,  $\beta^q \in C^q(\mathcal{L})$ , then  $\alpha^p \cup \beta^q \in C^{p+q}(\mathcal{L})$

$$\alpha^p \cup \beta^q([v_0, \dots, v_{p+q}]) := \alpha^p([v_0, \dots, v_p])\beta^q([v_p, \dots, v_{p+q}]).$$

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## Integral

Let  $C_\bullet$  be a chain complex. Let  $\alpha^p \in C^p$ ,  $c_p \in C_p$ , then set

$$\int_c \alpha^p := \langle c_p, \alpha^p \rangle = \alpha^p(c_p).$$

**Gauge-Fields:**

$$a(e_j) = z_j, b(e_j) = x_j$$

**Controlled Z:**

$$CCZ_{i,j,h} = (-1)^{a(i)a(j)a(h)}$$

Let  $\mathcal{L}$  be a  $D$ -dimensional simplicial complex.

Let  $\mathcal{C}, \mathcal{C}', \mathcal{C}''$  be CSS codes on  $\mathcal{L}$

(i.e.  $\mathcal{C}_1 = (\mathbb{Z}/2)\mathcal{L}_I$ )

such that  $I + I' + I'' = D$ . Let  $\Sigma \in (\mathbb{Z}/2)\mathcal{L}_D$ .

Set  $U := (-1)^{\int_{\Sigma} a \cup a' \cup a''}$ .

$\mathcal{C}_\bullet \in \text{Ch}(\mathbb{Z}/2)$  CSS  
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**Gauge-Fields:**

$$a(e_j) = z_j, b(e_j) = x_j$$

**Controlled Z:**

$$\text{CCZ}(\gamma_1, \gamma_2) = (-1)^{a(\gamma_1)a(\gamma_2)}$$

**Coboundary:**

$$d: C^j \rightarrow C^{j+1}$$

$$dc(\gamma) = \sum_{\sigma \in \partial \gamma} c(\sigma)$$

Let  $f \in \mathcal{C}_2$ . Then,

$$S_f^Z = \prod_{e \subset f} Z(e)$$

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Let  $v \in \mathcal{C}_0$ , then  $S_v^X = (-1)^{\sum_{e: \partial e \supset v} b(e)}$

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## Definition

For  $v \in \mathcal{C}_1$  and its dual  $v^* \in \mathcal{C}^1$  set  $\tilde{v} := |v\rangle\langle v|v^*$

## Fact

Let  $g$  be a polynomial. For any  $v \in \mathcal{C}_1$  and its dual  $v^* \in \mathcal{C}^1$ :

$$S_v^X (-1)^{g(a)} S_v^X = (-1)^{g(a+d\tilde{v})}.$$

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**Gauge-Fields:**

$$a(e_j) = z_j, b(e_j) = x_j$$

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$$dc(\gamma) = \sum_{\sigma \in \partial \gamma} c(\sigma)$$

**Stabilizer:**

$$S_f^Z = (-1)^{da(f)}.$$

$$S_v^X (-1)^{g(a)} S_v^X = (-1)^{g(a+dv^*)}$$

Let  $\mathcal{L}$  be a triangulation of a closed D-manifold  $M$   
and  $\mathcal{C}, \mathcal{C}', \mathcal{C}''$  suitable CSS codes on  $\mathcal{L}$ .

**Claim:**  $U = (-1)^{\int_M a \cup a' \cup a''}$  is a logical gate.

## Intersections

Let  $M$  be a closed  $D$ -manifold. Let  $\alpha \in H^p(M, \mathbb{Z}/2)$ ,  $\beta \in H^{D-p}(M, \mathbb{Z}/2)$ . Their Poincare-duals  $\alpha^*$  and  $\beta^*$  are  $p$  and  $D - p$ -codimensional submanifolds.

If  $\alpha^*$  and  $\beta^*$  intersect transversally, then

$$\int_M \alpha \cup \beta = [M] \cap (\alpha \cup \beta) = |\alpha^* \cap \beta^*| \bmod 2.$$

*Proof:* [Hut11]

### Gauge-Fields:

$$a(e_j) = z_j, b(e_j) = x_j$$

### Coboundary:

$$d: C^j \rightarrow C^{j+1}$$

$$dc(\gamma) = \sum_{\sigma \in \delta\gamma} c(\sigma)$$

### Operator cocycles

Let  $\alpha \in C^1$ .

$$\tilde{\alpha} := |\alpha\rangle\langle\alpha|$$

in  $C^1(C_\bullet, \mathfrak{u}(2^n))$ .

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More? Haruna - arXiv:2511.15224 [Har25].

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## Magic State

Let  $k := \dim H_1(\mathcal{C}_{\text{tot}}) = \dim H^1(\mathcal{C} \otimes \mathcal{C}' \otimes \mathcal{C}'')$ . Then,

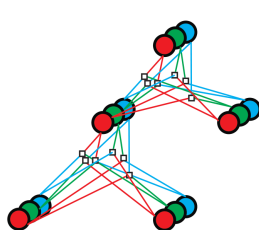
$U|+\rangle^{\otimes k} = \prod_{|\alpha^* \cap \beta^* \cap \gamma^*| = 1 \bmod 2} \overline{CCZ}(\alpha, \beta, \gamma) \overline{|+\rangle}^{\otimes k}$  is not a stabilizer state.

## Magic State

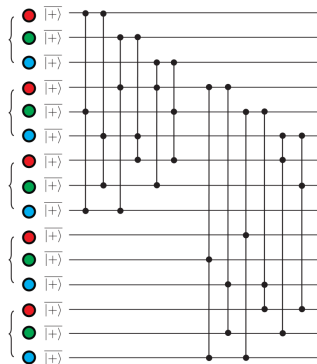
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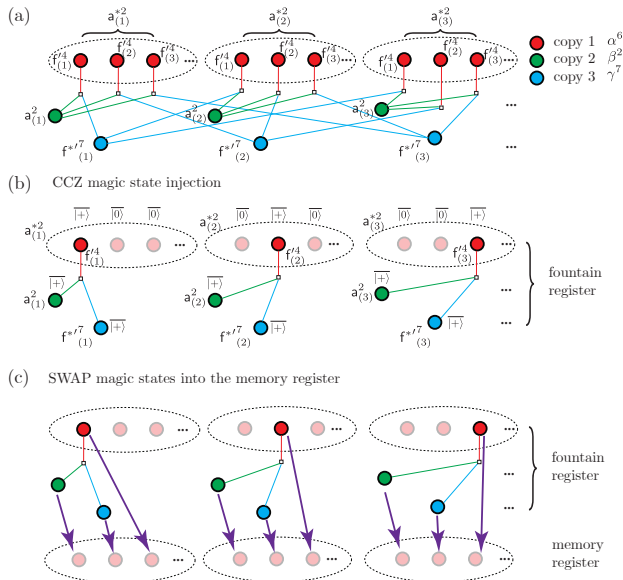
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For  $\mathcal{C} = \text{TC}_M^{\otimes 3}$



● copy 1  
● copy 2  
● copy 3





## Magic State

Let  $k := \dim H_1(\mathcal{C})$ . Then,

$$U\overline{|+\rangle}^{\otimes k} = \prod_{|\alpha^* \cap \beta^* \cap \gamma^*| = 1 \bmod 2} CCZ(\alpha, \beta, \gamma) \overline{|+\rangle}^{\otimes k}$$

is not a stabilizer state.

## Parallelization

Let  $\mathcal{C} = \text{TC}(M)^{\otimes 3}$ . Let  $\Upsilon := \#\text{non-overlapping intersections in } M$ . Then,  $\mathcal{C}$  can inject  $\Upsilon$  many non-transversal logical CZZ in parallel.

## Magic State

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## Parallelization

Let  $\mathcal{C} = \text{TC}(M)^{\otimes 3}$ . Let  $\Upsilon := \# \text{non-overlapping intersections in } M$ . Then,  $\mathcal{C}$  can inject  $\Upsilon$  many non-transversal logical CZZ in parallel.

Rowshan [Row26] classified magic state fountains algebraically

## Summary

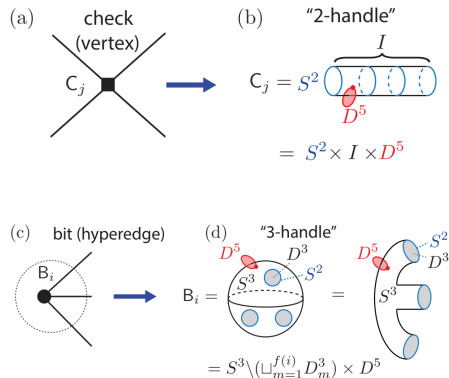
D-dimensional  $\mathcal{C}^\bullet$  needs:

non-trivial Cup-Products + Poincare-Duality +  $H^D(\mathcal{C}, \mathbb{Z}/2) = \mathbb{Z}/2$ .



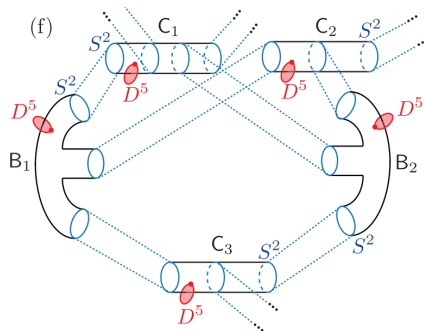
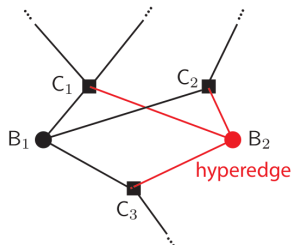
Let  $\overline{C} = \overline{C}_1 \xrightarrow{H} \overline{C}_0$  be a classical code. Consider its Tanner graph.

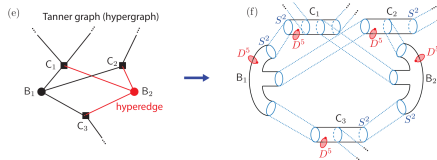
Let  $\overline{\mathcal{C}} = \overline{\mathcal{C}}_1 \xrightarrow{H} \overline{\mathcal{C}}_0$  be a classical code. Consider its Tanner graph.



Attach according to  $\bar{H}$ :

(e) Tanner graph (hypergraph)





Let  $\overline{C} = \overline{C}_1 \xrightarrow{H} \overline{C}_0$  be a classical code.

Let  $C: C_1 \xrightarrow{\bar{H}} C_0$  be its  $\mathbb{Z}$ -lift.

Then, the handle-decomposition of handle body  $X$  is

$$\begin{aligned} \dots \rightarrow \mathbb{Z}\{3\text{-handles}\} &\xrightarrow{\bar{H}} \mathbb{Z}\{2\text{-handles}\} \rightarrow \mathbb{Z}\{1\text{-handles}\} \rightarrow \mathbb{Z}\{0\text{-handles}\} \rightarrow 0 \\ &= \\ C_1 &\xrightarrow{\bar{H}} C_0 \end{aligned}$$

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 & & = & & & & = \\
 & & C_1 & \xrightarrow{\quad \bar{H} \quad} & C_0 & & 
 \end{array}$$

## Definition

$$M^8 := X \cup_{\partial X} X$$

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## Definition

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Then, the handle-decomposition of  $M^8$  is

$$0 \rightarrow \mathbb{Z}\{8\text{-h}\} \rightarrow \mathbb{Z}\{7\text{-h}\} \rightarrow C_0 \xrightarrow{H^T} C_1 \longrightarrow \dots$$

$$\dots \longrightarrow 0 \longrightarrow C_1 \xrightarrow{\overline{H}} C_0 \rightarrow \mathbb{Z}\{1\text{-h}\} \rightarrow \mathbb{Z}\{0\text{-h}\} \rightarrow 0$$

## Note

Triangulations of handles can be refined to triangulation of  $M^8$

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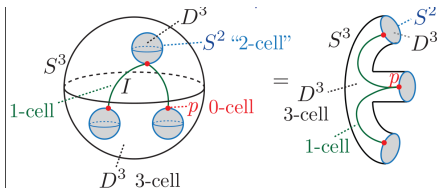
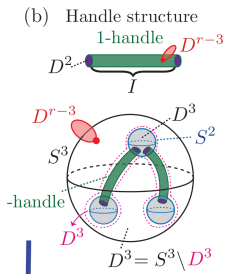
## Theorem

Let  $\bar{C} = \text{Ker}(\bar{H})$  be a LDPC code with parameters  $[n, k, d]$ . The 8-dimensional triangulated manifold  $M^8$  satisfies:

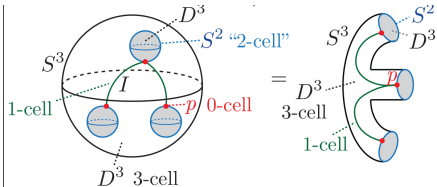
- ①  $M^8$  has a bounded local geometry,  
i.e., each vertex in its triangulation is adjacent to  $O(1)$  simplices.
- ②  $M^8$  contains  $\Theta(n)$  number of  $k$ -simplices ( $0 \leq k \leq r$ ),  
i.e.,  $\dim(C_k) = \Theta(n)$ , where  $C_k$  represents the  $k^{\text{th}}$  chain group.
- ③  $b_3 = \dim[H_3(M^8; \mathbb{Z}/2)] = k$ ;  
 $b_2 = \dim[H^2(M^8; \mathbb{Z}/2)] = k^T$
- ④  $\text{sys}_3(M^8; \mathbb{Z}/2) = \Omega(d)$ ,  $\text{sys}^2(M^8; \mathbb{Z}/2) = \Omega(d^T)$ ,  
where  $d$  and  $d^T$  are the distances of  $\bar{C}$  and  $\bar{C}^T$  respectively.

## Connectivity

Subdivision for handles can increase connectivity.



1



- "Handle" of Zhu have a CW-Structure.
- Attaching is a valid attaching for CW-complexes.

$M^4$

Good classical LDPC code  $\xRightarrow{\text{Zhu}} M^4$ .

$M^{11}$

Panteleev-Kalachev qLPDC code  $\xRightarrow{\text{F.H.}} M^{11}$ .

## $\sqrt{N}$ Distance

Let  $M^{15} := M^4 \times M^{11}$  and  $\mathcal{L}$  its triangulation. Define  $\mathcal{C}$  as the 3D-Product code of the CSS on the 6-, 2- and 7-th stage of  $\mathcal{L}$ . Then,

- $\mathcal{C}$  is a  $[[N, \Theta(N), \Omega(\sqrt{N})]]$  qLDPC code.
- $\mathcal{C}$  has constant stabilizer weight.
- $\mathcal{C}$  can inject  $\Theta(\sqrt{N})$  magic states in parallel (Single-shot).

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## $N^{2/3}$ Distance

Let  $M^{33} := M^{11} \times M'^{11} \times M''^{11}$ . Define  $\mathcal{C} := \text{TC}(M^{11})^{\otimes 3}$ . Then,

- $\mathcal{C}$  is a  $[[N, \Theta(N^{2/3}), \Omega(N^{2/3})]]$  qLDPC code.
- $\mathcal{C}$  has constant stabilizer weight.
- $\mathcal{C}$  can inject  $\Theta(N^{1/3})$  magic states in parallel (Single-shot).



Thanks for your attention!  
Any Questions?

- [Che+26] I-Chi Chen et al. *Efficient Magic State Factory Via Transversal Non-Clifford Gate*. 2026. arXiv: 2606.16199 [quant-ph]. URL: <https://arxiv.org/abs/2606.16199>.
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